Louisiana Believes

SAMPLE TEST ITEMS

Geometry

CONTENTS

INTRODUCTION

Louisiana Believes embraces the principle that all children can achieve at high levels and promotes the idea that Louisiana's educators should be empowered to make decisions to support the success of their students. In keeping with these values, the Louisiana Department of Education (LDOE) created this document with released and sample test items to help prepare teachers and students for the End-of-Course (EOC) assessments. These items reflect the LDOE's commitment to deliver consistent and rigorous assessments and provide educators and families with clear information about expectations for student performance.

Purpose of This Document

Teachers are encouraged to use the released and sample test items to gauge student learning, guide instruction, and develop classroom assessments and tasks. The document includes multiple-choice and constructed-response items that exemplify how the Louisiana Student Standards for Mathematics will be assessed on the Geometry EOC test. A discussion of each item highlights the knowledge and skills the item is intended to measure. It is important to remember that these sample items represent only a portion of the knowledge and skills measured by the Geometry EOC test. The items assembled in this document have been previously available in separate documents released in 2013-2014 and 2014-2015. Additionally, these items have been reviewed by the LDOE to confirm alignment to the newly adopted content standards.

MULTIPLE-CHOICE ITEMS

This section presents a sampling of multiple-choice items selected to illustrate the types of skills and knowledge students need in order to demonstrate understanding of the Louisiana Student Standards and Standards for Mathematical Practices in the Geometry course.

Information shown for each item includes the following:

- item data—conceptual category, domain, cluster, standard, Mathematical Practice(s) (MP), calculator designation (allowed or not allowed), correct answer
- commentary—on the skills and knowledge associated with the standard measured by the item, on the MP(s) linked with the item, on why the correct answer is correct (including how the answer is achieved), and on rationales for each incorrect answer option

GM: G-CO.D.12 Constructing a Perpendicular Bisector

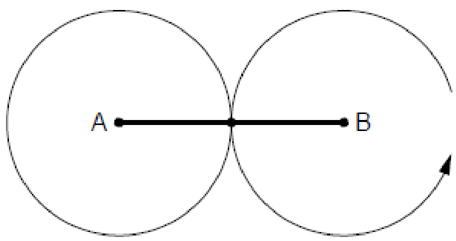
Make formal geometric constructions with a variety of tools and methods, e.g., compass and straightedge, string, reflective devices, paper folding, or dynamic geometric software. Examples: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Calculator Neutral

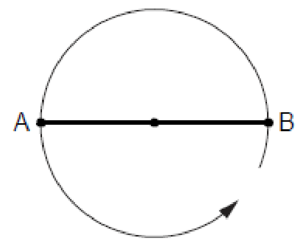
Melanie wants to construct the perpendicular bisector of line segment AB using a compass and straightedge.

Which diagram shows the first step(s) of the construction?

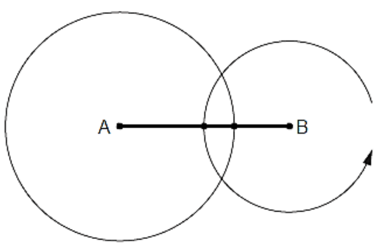
A.



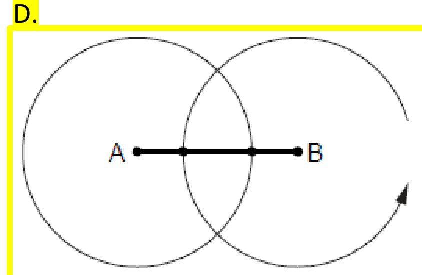
B.



C.



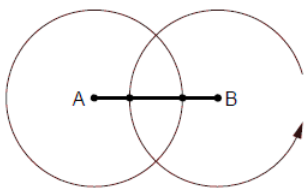
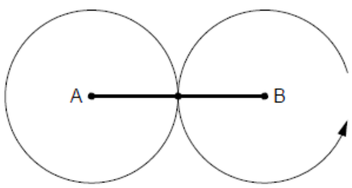
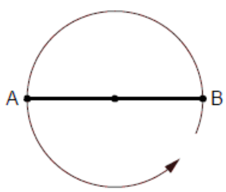
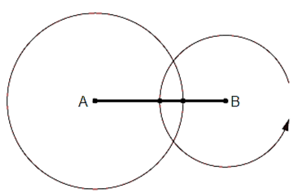
D.



This item requires students to identify the initial steps of constructing a perpendicular bisector. Constructions are typically difficult to assess in a multiple-choice format. However, asking students to analyze which figure will result in the given construction assesses reasoning about constructions without the need for providing geometric tools during testing or having to manually score each response.

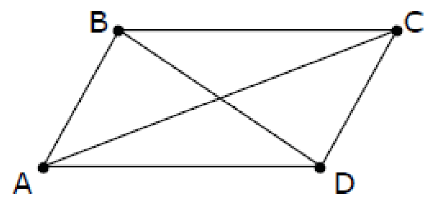
Mathematical Practice(s)

MP 5	Students must recognize how to effectively use a compass to begin constructing a perpendicular bisector of a line segment.
MP 6	Students must recognize that only one method shown guarantees a precise construction.
MP 7	Students must use the structure of the two congruent circles and recognize that the circles intersect at points that are equidistant from the endpoints of the given line segment. These points may be connected to construct a perpendicular bisector.

Correct Answer	Explanation
D. 	The figure shows two circles that appear to be congruent centered at the endpoints of the given line segment. Connecting the intersection points of these two circles guarantees the bisecting line created is perpendicular (at a 90 degree angle) to segment AB.
Incorrect Options	**Rationales for Incorrect Options**
A. 	This option has two circles that appear to be congruent and intersect at what appears to be the midpoint of segment AB; however, the figure does not show how a perpendicular bisector could be constructed, as it does not guarantee that a line drawn would intersect at a 90 degree angle.
B. 	This option has one circle that appears to be centered at the midpoint of segment AB. The midpoint is where a bisector will intersect, but this figure does not show how to ensure the bisector will be perpendicular.
C. 	This option has two circles with different radii centered at the endpoints of the given line segment. Connecting the intersection points of these two circles will result in a line perpendicular to segment AB that does not bisect it.

GM: G-CO.C.11 Opposite Sides of Parallelogram Proof
Prove and apply theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
Calculator Neutral

Use the figure to answer the question.



Missy is proving the theorem that states that opposite sides of a parallelogram are congruent.

Given: Quadrilateral ABCD is a parallelogram.
Prove: AB ≅ CD and BC ≅ DA

Missy's incomplete proof is shown.

	Statement		Reason
1.	quadrilateral ABCD is a parallelogram	1.	given
2.	$\overline{AB} \parallel \overline{CD}$; $\overline{BC} \parallel \overline{DA}$	2.	definition of parallelogram
3.	**?**	3.	**?**
4.	$\overline{AC} \cong \overline{AC}$	4.	reflexive property
5.	$\Delta ABC \cong \Delta CDA$	5.	angle-side-angle congruence postulate
6.	$\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$	6.	corresponding parts of congruent triangles are congruent (CPCTC)

Which statement and reason should Missy insert into the chart as step 3 to complete the proof?

A. BD ≅ BD; reflexive property
B. AB ≅ CD and BC ≅ DA ; reflexive property
C. ∠ABD ≅ ∠CDB and ∠ADB ≅ ∠CBD; when parallel lines are cut by a transversal, alternate interior angles are congruent
D. ∠BAC ≅ ∠DCA and ∠BCA ≅ ∠DAC; when parallel lines are cut by a transversal, alternate interior angles are congruent

This item requires students to provide the missing statement and reason in a two-column proof of the theorem that says that the opposite sides of a parallelogram are congruent. Assessing student's abilities to write proofs in a multiple-choice item is difficult. This item allows students to demonstrate their reasoning skills by asking for one of the middle steps in the proof, including both the statement and reason.

Mathematical Practice(s)	
MP 1	Students must analyze the given information in the item and the constraints of the details provided in the proof, and understand the goals of the problem.
MP 3	Students must complete the logical progression presented in the proof.

Correct Answer	Explanation
D. ∠BAC ≅ ∠DCA and ∠BCA ≅ ∠DAC; when parallel lines are cut by a transversal, alternate interior angles are congruent	Students are given a set of corresponding congruent sides in statement and reason 4. This leads into proving triangle congruence using the angle-side-angle congruence postulate in statement and reason 5. In order to reach statement and reason 5, students need to provide two pairs of corresponding congruent angles, which include the given corresponding congruent sides in statement and reason 4.
Incorrect Options	**Rationales for Incorrect Options**
A. BD ≅ BD; reflexive property	This answer is a true statement with a correct reason for the statement based on the given information; however, it is not relevant to completing the proof because it does not support statement and reason 5.
B. AB ≅ CD and BC ≅ DA; reflexive property	The student uses the final result of the proof as the statement and provides an incorrect reason for the statement.
C. ∠ABD ≅ ∠CDB and ∠ADB ≅ ∠CBD; when parallel lines are cut by a transversal, alternate interior angles are congruent	This response is also a true statement with a correct reason, but it is not relevant to the given proof because it does not support statement and reason 5.

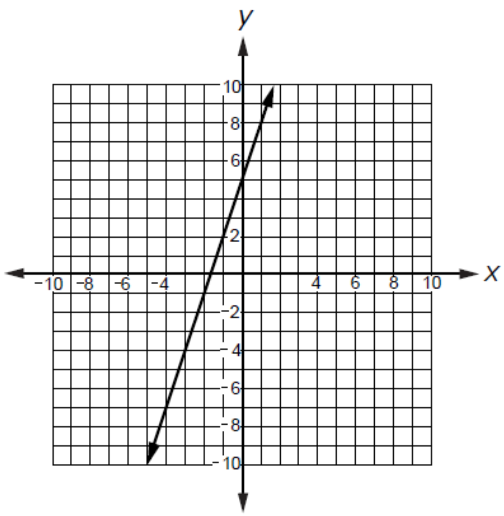
GM: G-SRT.A.1a Line Dilation Centered at (0, 7)

Verify experimentally the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

Calculator Not Allowed

Rosa graphs the line $y = 3x + 5$. Then she dilates the line by a factor of $\frac{1}{5}$ with (0, 7) as the center of dilation.



Which statement best describes the result of the dilation?

A. The result is a different line $\frac{1}{5}$ the size of the original line.

B. The result is a different line with a slope of 3.

C. The result is a different line with the slope of $-\frac{1}{3}$.

D. The result is the same line.

This item requires students to evaluate the effect of a dilation of a line not passing through the center of the dilation. Specifically, students must recognize that the dilated line will have the same slope as the original line.

Mathematical Practice(s)

MP 7 Students must recognize that the effect of the described dilation would not change the slope of the line, and that it would not be the same line either.

Correct Answer	Explanation
B. The result is a different line with a slope of 3.	The result of a dilation of a line not passing through the center of the dilation is a line parallel to the original line. The slope will remain the same.

Incorrect Options	Rationales for Incorrect Options
A. The result is a different line $\frac{1}{5}$ the size of the original line.	The student confuses the line with line segment. A dilation of a line segment with a scale factor of $\frac{1}{5}$ would result in a smaller line segment; however, a dilation of a line results in a line.
C. The result is a different line with the slope of $-\frac{1}{3}$.	The student incorrectly assumes that the result of the dilation is a line that is perpendicular to the original line.
D. The result is the same line.	The student incorrectly assumes that the result of the dilation is the original line; however, this is only true of lines that pass through the center of the dilation.

GM: G-SRT.B.5 Prove Triangles Similar
Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Calculator Neutral
Hector knows two angles in triangle A are congruent to two angles in triangle B. What else does Hector need to know to prove that triangles A and B are similar? A. Hector does not need to know anything else about triangles A and B. B. Hector needs to know the length of any corresponding side in both triangles. C. Hector needs to know all three angles in triangle A are congruent to the corresponding angles in triangle B. D. Hector needs to know the length of the side between the corresponding angles on each triangle.
This item requires students to determine which information is needed to complete an argument that two triangles are similar. Specifically, students must recognize that two pairs of corresponding congruent angles are sufficient for proving two triangles are similar. This item appears in the Online Tools Training (OTT).

Mathematical Practice(s)	
MP 1	Students must analyze the given information in the item and the constraints of the details provided, and understand the goals of the problem.
MP 3	Students must identify which additional piece of information, if any, is needed to construct an argument that establishes the similarity of the given triangles.

Correct Answer	Explanation
A. Hector does not need to know anything else about triangles A and B.	Two pairs of corresponding congruent angles are sufficient for proving two triangles are similar. Since the sum of the interior angles of all triangles is 180°, the third corresponding pair of angles of the two triangles must be the same measure.

Incorrect Options	Rationales for Incorrect Options
B. Hector needs to know the length of any corresponding side in both triangles.	The student incorrectly assumes that information about the corresponding sides of two triangles is necessary for proving similarity.
C. Hector needs to know all three angles in triangle A are congruent to the corresponding angles in triangle B.	The student incorrectly assumes that three pairs of corresponding congruent angles are necessary to prove two triangles are similar, which is not true because if two angles are known, then the third can be determined.
D. Hector needs to know the length of the side between the corresponding angles on each triangle.	The student incorrectly uses the angle-side-angle (ASA) criterion for congruence instead of AA criterion for similarity.

GM: G-GPE.A.1 Determine Center and Radius of a Circle	
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	
Calculator Not Allowed	
A circle has this equation. $x^2 + y^2 + 4x - 10y = 7$ What are the center and radius of the circle? A. center: (2, -5) radius: 6 B. center: (-2, 5) radius: 6	C. center (2, -5) radius 36 D. center (-2, 5) radius 36
This item requires students to use the method of completing the square in order to determine the center and radius of a circle given the equation.	

Mathematical Practice(s)	
MP 7	Students must complete the square for the equation to create an equivalent equation of the form $(x - h)^2 + (y - k)^2 = r^2$. They use this structure to determine that the center is (h, k) and the radius is r.

Correct Answer	Explanation
B. center: (-2, 5) radius: 6	Completing the square in the original equation results in $(x + 2)^2 + (y - 5)^2 = 6^2$. Therefore, the center of the circle is the point (–2, 5) and the radius is 6 units.

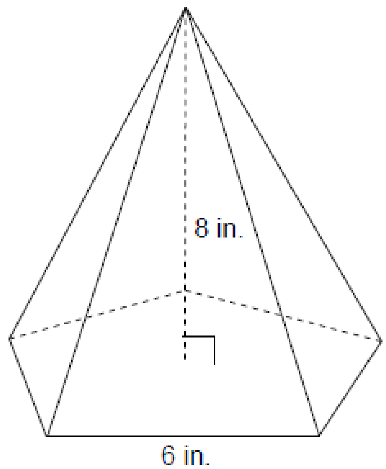
One possible method for solving is shown below.

step 1	$x^2 + 4x + y^2 - 10y = 7$	Rearrange *x*-terms together and *y*-terms together.
step 2	$(x^2 + 4x) + (y^2 - 10y) = 7$ $+4 \quad +25 \quad +4$ $+25$ $(x^2 + 4x + 4) + (y^2 - 10y + 25) = 36$	Complete the square for the expression $x^2 + 4x$ by adding 4 to both sides of the equation because $(4 \div 2)^2 = 4$. Complete the square for the expression $y^2 - 10y$ by adding 25 to both sides because $(-10 \div 2)^2 = 25$.
step 3	$(x + 2)^2 + (y - 5)^2 = 36$	Convert to factored form.
step 4	$(x - h)^2 + (y - k)^2 = r^2$ $(x - (-2))^2 + (y - 5)^2 = 6^2$ $h = -2 \quad k = 5 \quad r = 6$	Find the center at (h, k) and the radius at r.

Incorrect Options	Rationales for Incorrect Options
A. center: (2, -5) radius: 6	This response results from a sign error when finding the center of the circle in step 4.
C. center: (2, -5) radius: 36	This response is the result of a sign error when finding the center of the circle in step 4, and not taking the square root of the constant term to determine the radius in step 5.
D. center: (-2, 5) radius: 36	This response results from not taking the square root of the constant term to determine the radius in step 5.

GM: G-GMD.A.3 Volume of a Pentagonal Pyramid
Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
Calculator Allowed

Use the pyramid to answer the question.



This right pentagonal pyramid has a height of 8 inches and a base area of 61.94 square inches. To the nearest hundredth, what is the volume of the pyramid?

A. 80.00 cubic inches
B. 165.17 cubic inches
C. 240.00 cubic inches
D. 495.52 cubic inches

This item requires students to calculate the volume of a right pentagonal pyramid given the area of the base. This item appears in the Online Tools Training (OTT).

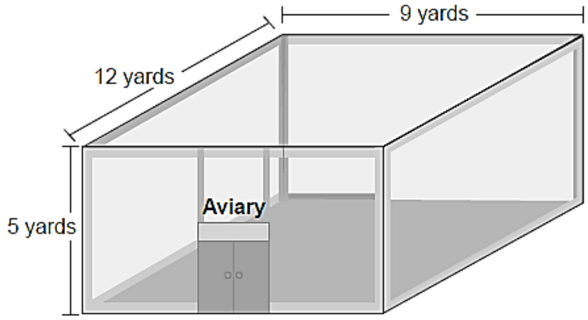
Mathematical Practice(s)	
MP 4	Students must identify the correct formula to calculate volume of a pentagonal pyramid. They must apply the given information.

Correct Answer	Explanation
B. 165.17 cubic inches	The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where *B* is the area of the base and *h* is the height. For this pyramid, $V = \frac{1}{3}(61.94)(8) = 165.17 \text{ cm}^3$.

Incorrect Options	Rationales for Incorrect Options
A. 80.00 cubic inches	This response is the result of multiplying $\frac{1}{3}$ by the product of the height, the given side length, and the number of sides of the base.
C. 240.00 cubic inches	This response results from computing the product of the height, the given side length, and the number of sides of the base.
D. 495.52 cubic inches	This answer results from failing to multiply $\frac{1}{3}$ by the product of the area of the base and the height.

GM: G-MG.A.2 Aviary Population Density
Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
Calculator Allowed

Use the diagram to answer the question.



An aviary is an enclosure for keeping birds. There are 134 birds in the aviary shown in the diagram.
What is the number of birds per cubic yard for this aviary? Round your answer to the nearest hundredth.

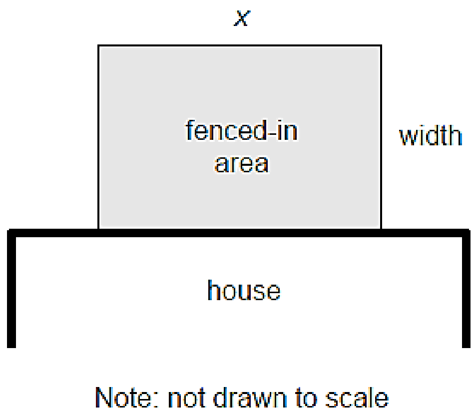
A. 0.19 birds per cubic yard
B. 0.25 birds per cubic yard
C. 1.24 birds per cubic yard
D. 4.03 birds per cubic yard

This item requires students to compute the volume of a rectangular prism. Then, students must use this value to determine the density of birds in an aviary.

Mathematical Practice(s)	
MP 1	Students must make sense of the given quantities and recognize that they must first compute the volume to then find the quotient of the number of birds and the total volume of the aviary.
MP 2	Students must decontextualize the given information and represent it in a logical mathematical format. Then, they must manipulate this representation and contextualize their results to answer the question.
MP 4	Students must model the given real-world situation using the equation for volume and the process to determine density.

Correct Answer	Explanation
B. 0.25 birds per cubic yard	The volume of the aviary is given by the product of 5 yards, 12 yards, and 9 yards. The density of birds per cubic yard is given by the quotient of the number of birds and the volume of the aviary.

Incorrect Options	Rationales for Incorrect Options
A. 0.19 birds per cubic yard	This response is the result of incorrectly using a sum of the given dimensions instead of calculating the volume of the aviary.
C. 1.24 birds per cubic yard	This answer results from incorrectly computing the number of birds per square yard of floor space.
D. 4.03 birds per cubic yard	This response is the result of calculating the number of cubic yards per bird rather than the number of birds per cubic yard.

GM: G-MG.A.3 Length of Fence for Maximum Area	
Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	
Calculator Allowed	
Use the diagram to answer the question.  Beth is going to enclose a rectangular area in back of her house. The house wall will form one of the four sides of the fenced-in area, so Beth will only need to construct three sides of fencing. Beth has 48 feet of fencing. She wants to enclose the maximum possible area. What amount of fence should Beth use for the side labeled *x*? A. 12 feet B. 16 feet C. 24 feet D. 32 feet	
This item requires students to solve an optimization problem involving area and perimeter. The most efficient pathway to solve the problem is to write equations to represent the area and perimeter. Then, students may solve for one variable in the linear perimeter formula and substitute this expression into the area formula. This will lead to a quadratic equation representing the area in terms of the length of one of the sides. Finding the maximum point of this parabola will determine the side length that will maximize the area.	
Mathematical Practice(s)	
MP 1	Students must make sense of the given quantities and recognize that they must compute both area and perimeter.
MP 2	Students must decontextualize the given information and represent it in a logical mathematical format. Then, they must manipulate this representation and contextualize their results to answer the question.
MP 4	Students must model the given real-world situation with equations for the area and perimeter.

Correct Answer	Explanation
C. 24 feet	Students recognize there are many possible lengths, but only one length will maximize the space available.

One possible method for solving is shown below.

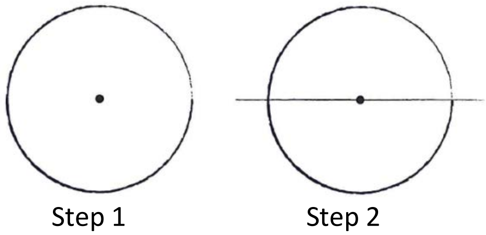
step 1	$x + 2w = 48$, where *w* represents the width $A = xw$	Determine necessary equations to use.
step 2	$w = \frac{48 - x}{2}$	Solve perimeter formula for *w*.
step 3	$A = x\left(\frac{48 - x}{2}\right)$	Replace *w* with the Area formula with equivalent expression from step 2.
step 4	$A = \left(-\frac{1}{2}\right)x^2 + 24x$	Simplify the expression.
step 5	$x = \frac{-b}{2a} = \frac{-24}{2 - \frac{1}{2}} = 24$	Find the *x*-value of the vertex of the parabola, which represents when the area is maximized.

Therefore, the area is maximized when $x = 24$ feet.

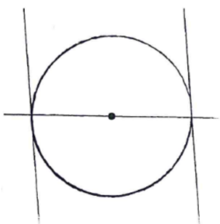
Incorrect Options	Rationales for Incorrect Options
A. 12 feet	This answer is a result of finding the width, *w*, for the maximized area instead of the length.
B. 16 feet	The student incorrectly assumes that a square will maximize the area of the fenced-in area without doing any calculations. This is true for situations where all four sides of the enclosed area are being used.
D. 32 feet	The student incorrectly assumes that the largest given value for *x* will result in the maximum area without doing any calculations.

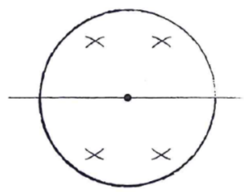
GM: G-CO.D.13 Square Inscribed in a Circle
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
Calculator Neutral

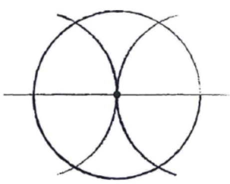
Use the diagram to answer the question.

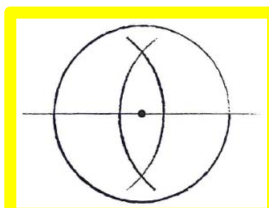


Daya is drawing a square inscribed in a circle using a compass and a straightedge. Her first two steps are shown. Which is the **best** step for Daya to do next?

A.


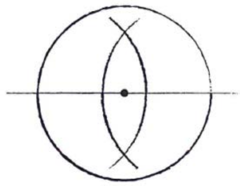
C.


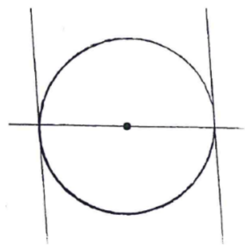
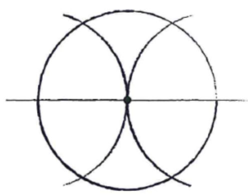
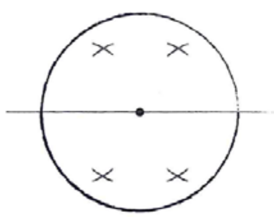
B.


D.


This item requires students to determine the subsequent steps to draw a square inscribed in a circle using only a compass and a straightedge. Since students cannot draw in the current EOC test environment, geometric constructions are best assessed as multiple-choice questions showing steps in the construction process. Students need to be familiar with the relationships between lines and circles and with various construction processes. This item appears in the Online Tools Training (OTT).

Mathematical Practice(s)	
MP 5	Students must recognize how to effectively use a compass and straightedge to complete the steps to construct a square inscribed in a circle.
MP 6	Students must recognize that only one method shown guarantees a precise construction.
MP 7	Students must use the structure of the two steps shown to choose the appropriate subsequent step.

Correct Answer	Explanation
D. 	Students recognize a perpendicular bisector must be drawn to establish the 90° angles required in a square. Option D and the preceding steps represent a standard construction process.[1]

Incorrect Options	Rationales for Incorrect Options
A. 	Students choosing this option demonstrate a lack of fundamental knowledge of the relationship between squares, circles, angles and lines. Option A does not consider the use of a compass at all, instead shows a step involving the drawing of tangent lines which are essentially useless and random.
B. 	Students may think that intersection points of the arcs and the circle represent vertices of a square. While this would certainly create a rectangle, option B would not result in congruent sides necessary for a square.
C. 	It is unclear in option C how the arcs are placed and how the compass is being used. This option gives no indication of precision for angle measures and side lengths.

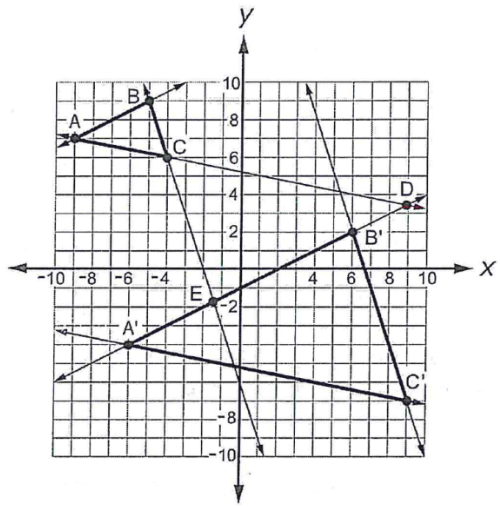
[1] A full demonstration of the process for constructing a square inscribed in a circle, including written directions and accompanying proof, can be found at http://www.mathopenref.com/constinsquare.html.

GM: G-SRT.A.3 Proving Angles Congruent for AA Similarity

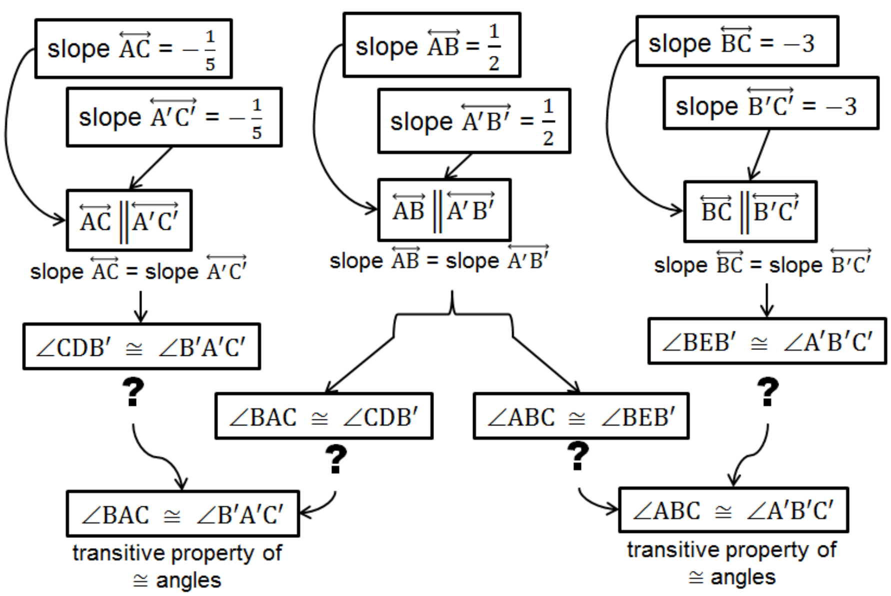
Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Calculator Neutral

Use the graph and flow chart to answer the question.



Kamal dilates triangle ABC to get triangle $A'B'C'$. He knows that the triangles are similar because of the definition of similarity transformations. He wants to demonstrate the angle-angle similarity postulate by proving $\angle BAC \cong \angle B'A'C'$ and $\angle ABC \cong \angle A'B'C'$. Kamal makes this incomplete flow chart proof.



What reason should Kamal add at all of the question marks in order to complete the proof?

A. Two non-vertical lines have the same slope if and only if they are parallel.
B. Angles supplementary to the same angle or to congruent angles are congruent.
C. If two parallel lines are cut by a transversal then each pair of corresponding angles is congruent.
D. If two parallel lines are cut by a transversal then each pair of alternate interior angles is congruent.

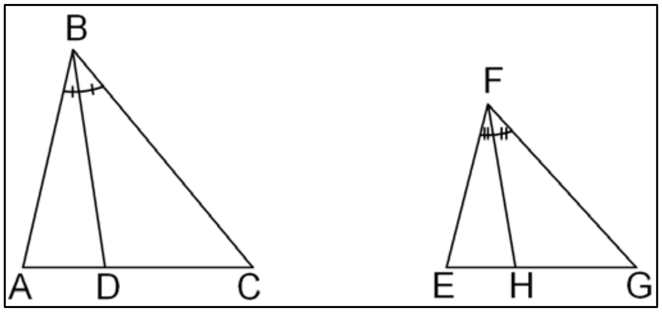
This item requires students to analyze the graph of several intersecting lines to determine the correct reasoning necessary to justify steps shown in a flowchart proof establishing AA criterion to prove congruence. This item appears in the Online Tools Training (OTT).	
Mathematical Practice(s)	
MP 1	Students must analyze the given information in the item and the constraints of the details provided in the proof and understand the goals of the problem.
MP 3	Students must complete the logical progression presented in the proof.
Correct Answer	**Explanation**
D. If two parallel lines are cut by a transversal then each pair of alternate interior angles is congruent.	Students recognize that option D is the only viable justification that applies to the four statements of congruence as each congruent pair are alternate interior angles.
Incorrect Options	**Rationales for Incorrect Options**
A. Two non-vertical lines have the same slope if and only if they are parallel.	This option does not justify angle congruence. It represents an earlier justification presented in the beginning steps of the proof.
B. Angles supplementary to the same angle or to congruent angles are congruent.	This option implies an existing congruence relationship that is not previously identified.
C. If two parallel lines are cut by a transversal then each pair of corresponding angles is congruent.	Students choosing this option may have mistaken the angles described as corresponding angles instead of alternate interior angles.

GM: G-SRT.B.4 Corresponding Angle Bisectors

Prove and apply theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity; SAS similarity criteria; SSS similarity criteria; ASA similarity.*

Calculator Neutral

Use the diagram and flow chart to answer the question.

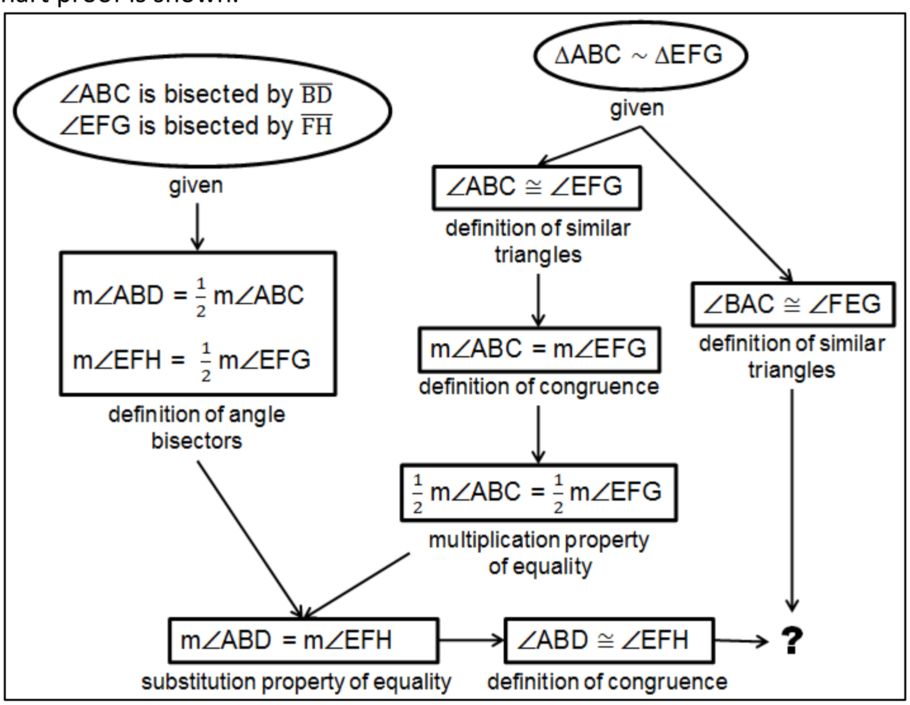


Ethan is proving the theorem that states that if two triangles are similar, then the measures of the corresponding angle bisectors are proportional to the measures of the corresponding sides.

Given: $\Delta ABC \sim \Delta EFG$; $\overline{BD}$ bisects $\angle ABC$, and $\overline{FH}$ bisects $\angle EFG$.

Prove: $\frac{AB}{EF} = \frac{BD}{FH}$

Ethan's incomplete flow chart proof is shown.



Which statement and reason should Ethan add at the question mark to **best continue** the proof?

A. $\Delta ABD \sim \Delta EFH$; AA similarity

B. $\angle BCA \cong \angle FGE$; definition of similar triangles

C. $\frac{AB}{BC} = \frac{EF}{GH}$; definition of similar triangles

D. $m\angle ADB + m\angle ABD + m\angle BAD = 180°$; $m\angle EFH + m\angle EHF + m\angle FEH = 180°$; Angle Sum Theorem

This item requires students to analyze the diagram and the incomplete proof given. Students use these tools to determine the appropriate next statement and justification to continue a triangle theorem proof.

Mathematical Practice(s)	
MP 1	Students must analyze the given information in the item and the constraints of the details provided in the proof and understand the goal of the problem.
MP 3	Students must continue the logical progression presented in the proof. This application is more challenging than items that require students to *complete* the proof because students cannot look to subsequent statements and justifications as additional cues to the logic presented.

Correct Answer	Explanation
A. $\Delta ABD \sim \Delta EFH$; AA similarity	Students recognize that the preceding statements and justifications lead to establishing similar triangles using AA similarity. Establishing triangle similarity for these two triangles is necessary to prove that the corresponding parts measures are proportional.

Incorrect Options	Rationales for Incorrect Options
B. $\angle BCA \cong \angle FGE$; definition of similar triangles	While true, it's not contributing to get closer to the goal of the proof.
C. $\frac{AB}{BC} = \frac{EF}{GH}$; definition of similar triangles	The equation in this option is not true regardless of the accompanying statement.
D. m$\angle$ADB + m$\angle$ABD + m$\angle$BAD = 180°; m$\angle$EFH + m$\angle$EHF + m$\angle$FEH = 180°; Angle Sum Theorem	Students choosing this option show a lack of fundamental understanding of the information given in the diagram and flowchart proof and/or of the process of completing a geometric proof.

GM: G-SRT.C.6 Triangle Sides Given Tangent Value

Understand that by similarity, side ratios in right triangles, including special right triangles (30-60-90 and 45-45-90), are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Calculator Allowed

Kendall drew a right triangle. The tangent value for one angle in her triangle is 1.8750. Which set of side lengths could belong to a right triangle similar to the triangle Kendall drew?

A. 16 cm, 30 cm, 35 cm
B. 8 cm, 15 cm, 17 cm
C. 6 cm, 8 cm, 10 cm
D. 1.875 cm, 8 cm, 8.2 cm

This item requires students to process the given information (tangent value, right triangle, similar triangles) in order to figure out which set of triangle side lengths will meet the given criteria without the benefit of a diagram. Students need to apply knowledge of how to determine if three side lengths form a right triangle, properties of similar triangles, how tangent values are calculated, and number sense. Note: Students may draw diagrams on their scratch paper.

Mathematical Practice(s)

MP 1 Students need to make sense of the given information in order to determine how to solve the problem.

Correct Answer	Explanation
B. 8 cm, 15 cm, 17 cm	This option is one of only two given that represents a right triangle. Determining whether a triangle is right or not with just the side lengths given can be found by applying the converse of the Pythagorean Theorem ($8^2 + 15^2 = 17^2$). Students recognize that tangent values will be the same in similar triangles because corresponding side lengths are proportional. In order to find a tangent value for an angle, students divide the opposite side length by the adjacent side length. Without the benefit of a diagram, students know that the longest side length is the hypotenuse, so they need to use the two shorter side lengths. Since the tangent value given (1.8750) is greater than one, students recognize that they are dividing a larger number by a smaller number, so 15 divided by 8.

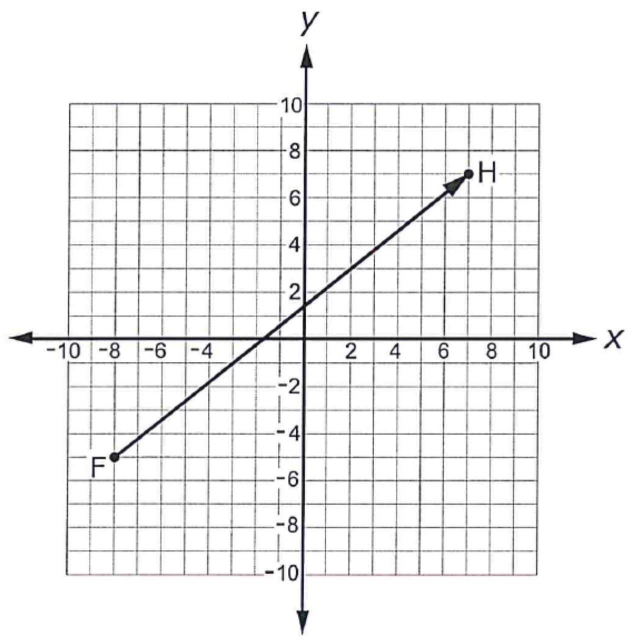
Incorrect Options	Rationales for Incorrect Options
A. 16 cm, 30 cm, 35 cm	This option is not a right triangle because $16^2 + 30^2 = 1156$ but $35^2 = 1225$. Students may have thought this was a correct answer because 30 divided by 16 equals 1.8750.
C. 6 cm, 8 cm, 10 cm	This option is the only other option besides the correct response that gives side lengths for a right triangle. Students may have chosen this option because the side lengths listed form a well-known Pythagorean triple. This is incorrect because 8 divided by 6 is $1.\overline{3}$.
D. 1.875 cm, 8 cm, 8.2 cm	This option is incorrect in two ways. First, it is not a right triangle because $1.875^2 + 8^2 = 67.515625$ but $8.2^2 = 67.24$. Students may have thought this was “close enough” to be a right triangle. Also, 8 divided by 1.875 does not equal 1.8750; it equals $4.2\overline{6}$. Most likely, students choose this option because one of the side lengths given is the same as the tangent value given.

GM: G-SRT.C.7 $\cos x° = \sin(90 - x)°$	
Explain and use the relationship between the sine and cosine of complementary angles.	
Calculator Not Allowed	
Adnan states if $\cos 30° \approx 0.866$, then $\sin 30° \approx 0.866$. Which justification correctly explains whether or not Adnan is correct? A. Adnan is correct because $\cos x°$ and $\sin x°$ are always equivalent in any right triangle. B. Adnan is correct because $\cos x°$ and $\sin x°$ are only equivalent in a $30° - 60° - 90°$ triangle. C. Adnan is incorrect because $\cos x°$ and $\sin(90 - x)°$ are always equivalent in any right triangle. D. Adnan is incorrect because only $\cos x°$ and $\cos(90 - x)°$ are equivalent in a $30° - 60° - 90°$ triangle.	
This item requires students to demonstrate conceptual understanding of the relationship between sine and cosine of complementary angles in order to determine the validity of a claim and select the appropriate justification for that reasoning.	
Mathematical Practice(s)	
MP 3 Students determine whether the given claim is correct and select justification to support the determination.	
Correct Answer	**Explanation**
C. Adnan is incorrect because $\cos x°$ and $\sin(90 - x)°$ are always equivalent in any right triangle.	Students determine that Adnan is incorrect because sine and cosine for a given angle are only equivalent in an isosceles right triangle where opposite and adjacent side lengths are the same value. Students justify their reasoning by recognizing that the relationship between sine and cosine of complementary angles is congruent. The adjacent side length used to calculate $\cos x°$ is the opposite side used to calculate $\sin(90 - x)°$. This value is the numerator and the hypotenuse length is the denominator for both trigonometric ratios.
Incorrect Options	**Rationales for Incorrect Options**
A. Adnan is correct because $\cos x°$ and $\sin x°$ are always equivalent in any right triangle.	This option is incorrect because sine and cosine are only equivalent in an isosceles right triangle where opposite and adjacent sides lengths are the same value. Students may misunderstand how cosine and sine are calculated.
B. Adnan is correct because $\cos x°$ and $\sin x°$ are only equivalent in a $30° - 60° - 90°$ triangle.	This option is incorrect because sine and cosine are only equivalent in an isosceles right triangle where opposite and adjacent sides lengths are the same value. Students may misunderstand how cosine and sine are calculated.
D. Adnan is incorrect because only $\cos x°$ and $\cos(90 - x)°$ are equivalent in a $30° - 60° - 90°$ triangle.	Students determine that Adnan is incorrect but select an incorrect justification. The justification in this option means the same as Adnan's claim. The value $\cos(90 - x)°$ is the same as $\sin x°$ in any right triangle. The value of $\cos x°$ does not equal the value of cosine of the complementary angle.

GM: G-C.A.1 Why are All Circles Similar?	
Prove that all circles are similar.	
Calculator Allowed	
Which statement explains why all circles are similar? A. There are 360° in every circle. B. The ratio of the circumference of a circle to its diameter is same for every circle. C. The diameter of every circle is proportional to the radius. D. The inscribed angle in every circle is proportional to the central angle.	
This item requires students to identify reasoning that proves why all circles are similar.	
Mathematical Practice(s)	
MP 8	Students should become familiar with this relationship after repeated calculation of circumference in class activities and tasks.
Correct Answer	**Explanation**
B. The ratio of the circumference of a circle to its diameter is same for every circle.	Students understand that the ratio of the circumference of a circle to its diameter is π because only π is multiplied by the diameter to find the circumference of a circle.
Incorrect Options	**Rationales for Incorrect Options**
A. There are 360° in every circle.	The sum of the degree measures in a figure does not indicate anything with regards to similarity. For example, the sum of the angles in any quadrilateral is equal to 360°, but a square does not have to be similar to a rhombus.
C. The diameter of every circle is proportional to the radius.	In order for two shapes to be proven similar using proportions, a proportion involving a corresponding side (circumference for a circle) must be used. While this option is a true statement, it does not prove similarity.
D. The inscribed angle in every circle is proportional to the central angle.	In order for two shapes to be proven similar using proportions, a proportion involving a corresponding side (circumference for a circle) must be used. While this option is a true statement, it does not prove similarity.

GM: G-GPE.B.6 Partition Segment 5 to 1
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Calculator Allowed

Use the graph to answer the question.



Point G is drawn on the line segment so that the ratio of FG to GH is 5 to 1. What are the coordinates of point G?

A. (4, 4.6)
B. (4.5, 5)
C. (−5.5, −3)
D. (−5, −2.6)

This item requires students to find the point (4.5, 5) on directed line segment FH that partitions the segment 5 to 1 by incorporating knowledge of translation, dilation factor, and proportional reasoning.

Mathematical Practice(s)	
MP 1	Students must understand the relationship between the points on the directed line segment and analyze the given constraints in order to solve the problem.

Correct Answer	Explanation
B. (4.5, 5)	Students recognize that a ratio of 5 to 1 would partition the line into 6 parts which would involve dilating the directed line segment by a factor of $\frac{5}{6}$.

One possible method for solving is shown below.

step 1	$x_2 - x_1$ $7 - (-8) = +15$	$y_2 - y_1$ $7 - (-5) = +12$	determine the translation rule from beginning (x_1, y_1) point F(−8, −5) to ending (x_2, y_2) point H (7, 7); +15 is a movement of 15 units to the right and +12 is a movement of 12 units up
step 2	$15 \times \frac{5}{6} = 12.5$	$12 \times \frac{5}{6} = 10$	multiply the translation numbers (15 and 12) by the dilation factor $\left(\frac{5}{6}\right)$; a ratio of 5 to 1 would mean a partition into 6 parts and 5 of 6 parts would be $\frac{5}{6}$
step 3	$(-8) + 12.5 = 4.5$	$(-5) + 10 = 5$	add the dilated lengths (12.5 and 10) to the original coordinates of point F(−8, −5)

Point G will be located at (4.5, 5).

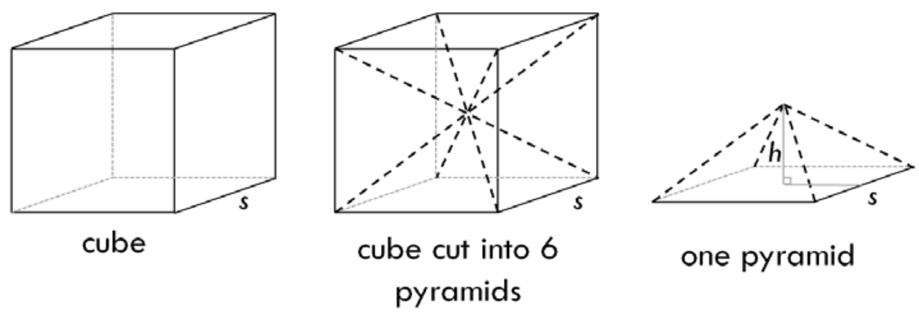
Incorrect Options	Rationales for Incorrect Options
A. (4, 4.6)	Students choosing this option may have thought that a 5 to 1 ratio meant a partition into 5 parts. Option A represents multiplying by a factor of $\frac{4}{5}$ in step 2.
C. (−5.5, −3)	Students choosing this option may have swapped F and H as beginning and end points or calculated a 1 to 5 ratio instead. Option C represents changing the order of subtraction in step 1 or multiplying by a factor of $\frac{1}{5}$ in step 2.
D. (−5, −2.6)	Students choosing this option may have thought that a 5 to 1 ratio meant a partition into 5 parts. Option D represents this misconception and an additional error of either changing the order of subtraction in step 1 **or** multiplying by a factor of $\frac{1}{4}$ in step 2.

GM: G-GMD.A.1 Deriving Pyramid Volume Formula

Give an informal argument, e.g., dissection arguments, Cavalieri's principle, or informal limit arguments, for the formulas for the circumference of a circle; area of a circle; volume of a cylinder, pyramid, and cone.

Calculator Allowed

Use the figures to answer the question.



Sasha derived the formula for the volume of a square pyramid. She started by dividing a cube into 6 identical square pyramids. The top vertex of each pyramid meets at the central point in the cube, with the cube's diagonals as the edges.

$V =$ the volume of a pyramid $\quad s =$ side length of base $\quad h =$ height of pyramid.

The steps of Sasha's work are shown.

Step 1: $6V = s^3$ $\qquad$ Step 2: $V = \frac{1}{6}s^3$

Maggie also derived the formula for volume of a square pyramid. Maggie's result is $V = \frac{1}{3}s^2h$. The formulas derived by Sasha and Maggie can both be used to correctly calculate the volume of a square pyramid. What are the **best** next steps for Sasha to take to prove that either formula can be used to find the volume of a square pyramid?

A.

step 3	$2h = s$
step 4	$V = \frac{1}{6}(2h)^3$
step 5	$V = \frac{1}{3}8h^3$

B.

step 3	$2h = s$
step 4	$V = \frac{1}{6}s^2(s)$
step 5	$V = \frac{1}{6}s^2(2h)$

C.

step 3	$2s = h$
step 4	$s = \frac{1}{2}h$
step 5	$V = \frac{1}{6}s^2(s)$
step 6	$V = \frac{1}{6}s^2\left(\frac{1}{2}h\right)$

D.

step 3	$2s = h$
step 4	$s = \frac{1}{2}h$
step 5	$V = \frac{1}{6}\left(\frac{1}{2}h\right)^3$
step 6	$V = \frac{1}{6}\left(\frac{1}{8}\right)h^3$

This item requires students to analyze the given diagrams and the algebraic work shown in order to continue the derivation process. While students have the formula to find volume of a square pyramid on their Geometry Reference Sheet, the formula shown on the sheet is of a different form than either formula given in the problem.[2]	
Mathematical Practice(s)	
MP 1	Students need to make sense of the given diagram, context, and calculations in order to determine how to connect one equation to the other.
MP 4	Students need to model the geometric relationship shown with algebraic computations.
MP 6	Students need to examine the answer choices closely to determine which shows the precise derivation that would follow from the given steps and is free from mathematical errors.
MP 7	Students consider the physical structure of the diagram and the structure of the steps shown to determine which set of continuing steps follow the same structure.

Correct Answer	Explanation
B.	Students recognize that multiplying the height of the pyramid by 2 will equal the side length because the faces of a cube are all squares. For step 4, students go back to step 2 and recognize that s^3 is equal to $s \times s \times s$ or $s^2(s)$. This form is important because s^2 is the area of the base. In step 5, students take the knowledge from step 3 to replace (s) with $(2h)$. This sets up for the remainder of the steps necessary to complete the process. For a sixth step, students could simplify by multiplying $2 \times \frac{1}{6}$ to get $\frac{1}{3}$.

step 3	$2h = s$
step 4	$V = \frac{1}{6}s^2(s)$
step 5	$V = \frac{1}{6}s^2(2h)$

[2] The formula for volume of a square pyramid shown on the Geometry Reference Sheet is $V = \frac{1}{3}Bh$ where *B* represents the area of the base of the pyramid.

<table>
<tr><th>Incorrect Options</th><th>Rationales for Incorrect Options</th></tr>
<tr><td>A.
<table>
<tr><td>step 3</td><td>$2h = s$</td></tr>
<tr><td>step 4</td><td>$V = \frac{1}{6}(2h)^3$</td></tr>
<tr><td>step 5</td><td>$V = \frac{1}{3}8h^3$</td></tr>
</table></td>
<td>Option A starts the same as correct option B by recognizing that two heights equal the length of one side. Step 4 is a legitimate route to take by replacing s^3 with $(2h)^3$. The error is in step 5; the 2 is used when simplifying, $2 \times \frac{1}{6} = \frac{1}{3}$. However, $2 \times \frac{1}{6}$ cannot be simplified before the expression $(2h)^3$ is evaluated because in the order of operations evaluating exponents is done before multiplication. The expression $(2h)^3$ is correctly evaluated as $8h^3$. Simplifying would then be $8 \times \frac{1}{6} = \frac{8}{6} = \frac{4}{3}$.</td></tr>
<tr><td>C.
<table>
<tr><td>step 3</td><td>$2s = h$</td></tr>
<tr><td>step 4</td><td>$s = \frac{1}{2}h$</td></tr>
<tr><td>step 5</td><td>$V = \frac{1}{6}s^2(s)$</td></tr>
<tr><td>step 6</td><td>$V = \frac{1}{6}s^2\left(\frac{1}{2}h\right)$</td></tr>
</table></td>
<td>In option C, the equation in step 3 is incorrect which results in the remaining steps being incorrect.</td></tr>
<tr><td>D.
<table>
<tr><td>step 3</td><td>$2s = h$</td></tr>
<tr><td>step 4</td><td>$s = \frac{1}{2}h$</td></tr>
<tr><td>step 5</td><td>$V = \frac{1}{6}\left(\frac{1}{2}h\right)^3$</td></tr>
<tr><td>step 6</td><td>$V = \frac{1}{6}\left(\frac{1}{8}\right)h^3$</td></tr>
</table></td>
<td>In option D, the equation in step 3 is incorrect which results in the remaining steps being incorrect.</td></tr>
</table>

GM: G-MG.A.1 Hemisphere to Model Shape of a Pond
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Calculator Allowed

Use the diagrams to answer the question.

Diagram 1: Side view of City Park Pond

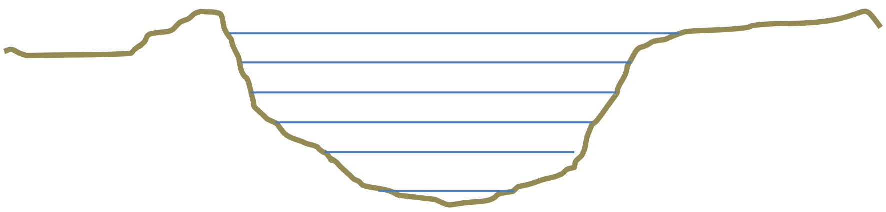
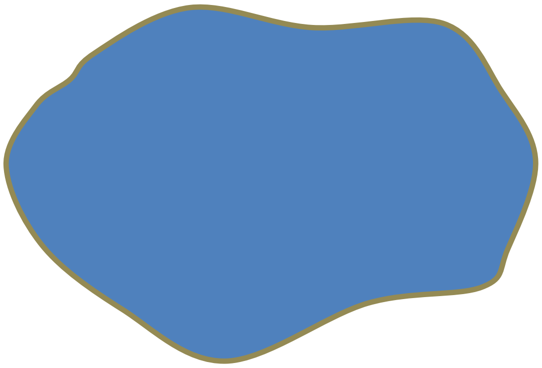


Diagram 2: Top view of City Park Pond



Based on the two diagrams shown, which formula would be **best** to use to estimate the volume of City Park Pond?

A. $V = \pi r^2 h$

B. $V = \frac{2}{3} \pi r^3$

C. $V = \frac{1}{3} Bh$

D. $V = \frac{1}{3} \pi r^2 h$

This item requires students to determine the appropriate 3-dimensional figure to model the context and identify which volume formula corresponds to that figure. The formula for volume of a hemisphere is not on the Geometry Reference Sheet. Students need to derive this formula by taking half of the formula for volume of a sphere, which is on the reference sheet.

Mathematical Practice(s)	
MP 4	Students need to model the shape of the pond with the most appropriate 3-dimensional figure.
MP 7	Students need to analyze the physical structure shown in the two diagrams.

Correct Answer	Explanation
B. $V = \frac{2}{3}\ \pi r^3$	Students correctly determine that the best 3-dimensional shape to model the pond is a hemisphere and identifies the formula for volume of a hemisphere. Students should recognize that a hemisphere is half a sphere so the volume of a hemisphere would be half the volume of a sphere $\left(V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right)$.
Incorrect Options	**Rationales for Incorrect Options**
A. $V = \pi r^2 h$	Students determine the shape to be cylindrical which would not cover as much volume as a hemisphere, and so is not the **best** option.
C. $V = \frac{1}{3} Bh$	Students determine the shape to be pyramid-like or conic which would not cover as much volume as a hemisphere, and so is not the **best** option.
D. $V = \frac{1}{3}\pi r^2 h$	Students determine the shape to be conic which would not cover as much volume as a hemisphere, and so is not the **best** option.

CONSTRUCTED-RESPONSE ITEMS

This section presents three constructed-response items, scoring information, and samples of student responses (for two of the three items) that received scores of 4, 3, 2, 1, 1 for minimal understanding, and 0.

GM: G-MG.A.3 Soybean Yield

Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Calculator Allowed

Mr. Fontenot planted four types of soybeans on his land in order to compare overall cost (for planting and harvesting) and crop harvest. The table shows the number of acres planted, the cost per acre, and the number of bushels of soybeans produced for the different types of soybeans.

Type of Soybean	Number of Acres Planted	Cost (per acre) to Harvest	Number of Bushels Produced
A	200	$174.70	9,000
B	150	$180.90	7,500
C	100	$192.40	5,900
D	75	$204.00	4,500

Part A

Regulations specify that Mr. Fontenot cannot devote more than 80% of a field to one particular type of soybean. He wants to design a field so that he can harvest the most soybeans for the lowest cost. What is the best design plan for Mr. Fontenot's 525 acres? Include specific details about which soybeans you chose, how many acres of each type should be planted, and why you chose those soybeans.

Part B

This table shows the profit Mr. Fontenot can earn per bushel for each type of soybean.

Type of Soybean	Profit per Bushel
A	$4.50
B	$3.88
C	$3.96
D	$4.24

Determine if the design plan created in part A is the most profitable 80/20 design.

- If part A is the most profitable plan, explain why it is the most profitable and include specific details about the profitability of the plan from part A compared to all other possible design plans.
  OR
- If part A is **not** the most profitable plan, determine which design plan is the most profitable and include specific details about the profitability of the plan from part A compared to this design plan.

This item requires students to analyze the given information in order to determine the **best** design plan to meet the given constraints in part A. Students also evaluate the determined design plan in light of new constraints presented in part B. Sufficient support/justification is required in both parts of the item.

Mathematical Practice(s)

MP 1	Students must examine and make sense of all of the given information in the problem and develop a solution pathway in order to provide the requested information.
MP 3	Students must construct viable arguments to support their reasoning.
MP 6	Students must complete the steps to solving with precision in order to determine the correct solutions and be mindful of their use of units throughout. Any calculations shown must be free of mathematical errors.

Scoring Information

This section includes information used to score this constructed-response item: an exemplary response, an explanation of how points are assigned, and a scoring rubric. Appropriate scoring parameters for all EOC constructed-response items are determined by Rangefinding Committees comprised of teachers and curriculum experts from across the state of Louisiana.

Scoring Rubric	
4	The student earns 4 points.
3	The student earns 3 points.
2	The student earns 2 points.
1	The student earns 1 point OR demonstrates minimal understanding of the standard being measured.
0	The student's response is incorrect, irrelevant to the skill or concept being measured, or blank.

Sample Answer

Part A

Mr. Fontenot should plant 420 acres of soybean C and 105 acres of soybean D.

525 * 0.8 = 420 Cost per bushel: A = $3.88; B = $3.62; C = $3.26; D = $3.40

Soybean C has the lowest cost per bushel to produce and therefore should be planted on the maximum 80%. Soybean D has the next lowest cost per bushel to produce and should be used for the other 20%.

Part B

The design plan in part A is not the most profitable 80/20 design. Mr. Fontenot should plant 420 acres of soybean D and 105 acres of soybean C. Based on the numbers of bushels per acre and the profit per bushel, Soybean D yields the greatest profit for the larger section of 420 acres. Soybean C yields the next greatest profit based on bushels per acre and profit, and should be used for the other 20%. This would be a total profit of $131,380.20 which is $6,539.40 greater than the profit of $124,840.80 from the plan in part A.

Points Assigned

Part A

2 points:

- 1 point for correct design of 420 acres of C and 105 acres of D
- 1 point for providing complete and correct support for their selection

Part B

2 points:

- 1 point for correct design of 420 acres of D and 105 acres of C
- 1 point for providing complete and correct support for their selection

Sample Student Responses[3]

Score Point 4

The following authentic student responses show the work of two students who each earned a score of 4. A score of 4 is received when a student completes all required components of the task and communicates his or her ideas effectively. The response should demonstrate in-depth understanding of the content objectives, and all required components of the task should be complete.

Score Point 4, Student Response 1
Part A Of the four types of soybeans, type C costs the least to produce per bushel at $3.26per bushel, followed by type D at $3.40 per bushel, type B at $3.62 per bushel, and finally, type A, the most expensive, at $3.88 per bushel. Since Mr. Fontenot can not plant more than 80% of his field with one bean, he should plant the fist 80% with tyoe C, and the remaining 20% with type D, which was the next most cost-effective soybean. This would mean Mr. Fontenot would be planting 420 acres of type C soybeans and 105 acres of type D soybeans. **Part B** Assuming that the profits listed take into account the cost of harvest, the plan created in part A would not be the most profitable. While type C soybeans are the least costly to harvest, type D soybeans grow more soybeans per acre, meaning more profits. The design of plan A would mean $124,840.80 in profit. If Mr. Fontenot planted instead 420 acre of Type D and 105 of Type C soybeans, however, the total profit would be $131,380.20. A total of $$6,539.40 more in profit than the field design from part A. The mostprofitable field design would be to plant 420 acres of Type D (80%)and 105 acres of Type C beans (20%).
This student response is correct and clear. In part A, the student provides the correct 80/20 partition and provides sufficient justification for their reasoning, including cost per bushel of all types and the number of acres of each type in the most profitable design plan. In part B, the student provides a correct design plan and sufficient support for their conclusion, including an analysis of the previous design plan provided in part A.

Score Point 4, Student Response 2
Part A The best design plan for Mr. Fontenot to use is to plant 420 acres of type C soybeans and 105 acres of type D soybeans. I chose to use the most of type C soybeans because it costs lowest price to harvest. It only cost $3.26 to harvest one bushel. I chose type D soybeans for the remainder of the field because it costs the second lowest price to harvest. It only costs $3.40 to harvest one bushel. **Part B** The design created in part A is not the most profitable 80/20 design. The profitability of the plan from part A is only $124840.80. Though the profitability would be higher if the plan was 420 acres of type D and 105 acres of type C. The profitability would then be $131380.20. The profitability of the new plan would make $6539.40 more than the profitability of the plan from part A.
This response receives full credit. In part A, the student provides the correct design plan and sufficient support for that plan. Although the student did not list the cost per bushel for each type of soybean, the response indicates that this process was completed (“it costs the lowest price” and “the second lowest price”). In part B, the student provides the correct design plan and sufficient support, including an analysis of the previous plan in part A and a comparison of the two design plans (“the new plan would make $6539.40 more”).

[3] All student responses are authentic student work and not edited in any way, so responses may include typographical errors such as misspelled words or missing spaces.

Score Point 3

The following authentic student responses show the work of two students who each earned a score of 3. A score of 3 is received when a student completes 3 of the 4 components correctly. There may be simple errors in calculations or some confusion with communicating his or her ideas effectively.

Score Point 3, Student Response 1
Part A Mr. Fontenot wants to find the most cost efficient and profitable scheme for harvesting his soybeans. The most profitable plan would be to have 420 feild (80%) of soybean C and 105 feild (20%) of soybean D. The total cost of this would be $102,228 which is more than the above charts total of $96,615. But this scheme is allowing you to grow more soybeans. This scheme allows you to grow 31,080 soybeans total while the chart above only allows you to grow 26,900 total soybeans. If you divide the total cost by the total number of bushles then you realize that my scheme only cost about $3.289 per bushle while the chart above cost about $3.596. So if you change your ploting stratagey to this one you could actually continue to sell the beans at the same price as before and make a larger profit with out having to raise your prices. **Part B** The plan I created would have been the most profitable with out knowing the profits of each bushel. Now knowing the profits earned each diffrent bushle I can conclude that 420 acres of soybean D and 105 acres of soybean C would be more profitable than my last plan in part A. My plan in part A, now knowing the actuall profits, would earn $124,840.80 in profits. The newest plan I just explained in Part B would earn $131,380.20 in profits. An extra profit of $6,539.40
In part A, the student provides a correct design plan but does not include sufficient support for this selection. Sufficient support needs to include a comparison to cost per bushel of soybean types A and B. In part B, the student provides a correct design plan and sufficient support, including an analysis of the previous plan and a comparison of the two plans ("An extra profit of $6,539.40."). The student earns 3 points for providing plans for both parts and one correct and complete justification (part A).

Score Point 3, Student Response 2
Part A I chose soybeans c and d because they had the lowest cost per bushel to harvest. Soybean C cost 3.261 $ to harvest and soybean D cost 3.4 $ to harvest. Whereas Soybean A cost 3.88 and soybean B cost 3.618 to harvest. **Part B** No this plan is not the most profitable 80/20 design, because per bushel soybean D is more profitable, so therefore there should be 420 acres of soybean D and 105 acres of soybean C.
In part A, the student correctly identifies soybeans C and D but does not indicate how much of each to plant (i.e., 420 acres of C and 105 acres of D). The support provided in part A is sufficient because it includes the cost per bushel of each type. In part B, the student provides the correct design plan and the justification provided is adequate ("per bushel soybean D is more profitable" implies a comparison of the two design costs). The student earns 3 points for providing one complete and correct plan (part B) and justification for both plans.

Score Point 2

The following authentic student responses show the work of two students who each earned a score of 2. A score of 2 is received when a student completes 2 out of 4 components correctly. There may be simple errors in calculations, one or two missing responses, or unclear or incorrect communications of his or her ideas.

Score Point 2, Student Response 1
Part A The first thing we want to do is to find the most effective soybean type and to do that we need to find the soybean type that has the largest yield per acre, so, let's simplify these fractions. (9000)/(200)=45 bushels per acre (7500)/(150)=50 bushels per acre (5900)/(100)=59 bushels per acre (4500)/(75)=60 bushels per acre The next step is to finds 80% of 525 acres so we can see how many acres we can devote the most effective soybean to. .80x525=420 acres The next step is to tind the most COST effective soybean type. Soybean D $204 x 420 acres =$85,680 420 acres x 60 bushels=25,200 bushels Cost per bushel = $3.40 per bushel Soybean C $192.40 x 420 acres = $80,808 420 acres x 59 bushels = 24,780 bushels Cost per bushel= $3.26 per bushel Soybean B $180.90 x 420 acres = 75978 420 acres x 50 bushels = 21,000 bushels Cost per bushel = $3.62 per bushel Soybean A $174.70 x 420 acres = $73,374 420 acres x 45 bushels= 18,900 bushels Cost per bushel = $3.88 per bushel So, even though Soybean D is the most expensive, it does yield the most so that is the soybean that will take up 80% of the field. The next step is to use the information above to determine which soy bean will take up the remaing space of 105 acres. Soybean C is the second greatest yielding plant so we'll use that. And now for the sake of wrapping it all up the grand total price for harvesting this field would be $105,882 The total amount of soybean bushels would be $31,395, which averages out to cost about $3.37 per bushel. **Part B** The profit Mr. Fontenot would recieve from the above plan is $131,380.20, i believe this to be the most profitable method because it is the plan with the largest yield.
In part A, the student provides an incorrect design plan, but the support provided is correct and includes the cost per bushel of each soybean type thus earning 1 point. In part B, the student correctly identifies the most profitable design plan but does not provide sufficient support. Sufficient support must include a comparison to at least one other plan to ensure that some analysis was completed by the student. The student earns 2 points for providing correct and complete support for part A and a correct design plan for part B.

Score Point 2, Student Response 2
Part A He should plant soybean type C with about 420 acres because soybean type C has the cheapest bushel per acre. The other 105 acres should be planted with soybean type D because it has the second cheapest bushel per acre. **Part B** The design plan in part A is the most profitable 80/20 design plan since soybean C has the lowest cost per bushel. However, soybean C has the second highest profit per bushel. Soybean type D has the second lowest cost per bushel, but has the highest profit per bushel for the design plan in part A.
This response receives two points in part A for the correct design plan and adequate support. An ideal response for part A includes the cost per bushel for types C and D, but the comparison language provided ("cheapest" and "second cheapest") implies that the calculations were done by the student and is adequate. For part B, the student chooses the plan from part A, which is incorrect. The student mentions that types C and D have the "second highest" and "highest" profits per bushel but gives no indication as to the relevance of that information and does not seem to use that information in plan analysis. The student earns 2 points for a correct design plan and correct and complete explanation in part A.

Score Point 1

The following authentic student responses show the work of two students who each earned a score of 1 for their responses. A score of 1 is received when a student completes 1 out of 4 components correctly.

Score Point 1, Student Response 1
Part A The best design plan for Mr. Fontenot's 525 acres would be to plant 105 acres of soybean "D"and 420 acres of soybean "C". I know this because soybeans "D" and "C" produce the highest percentages of bushels for the lowest cost per acre (29% and 30%). Whereas soybeans "A" and "B" produced the lowest percentages of bushels for a high cost per acre (25% and 27%) **Part B** The design plan created in part A is not the most profitable 80/20 design. The most profitable design would be to use soybeans "A" and "D". Soybeans "A" and "D" ($8.74) would bring in a higher profit than soybeans "C" and "D" ($8.20). (The cost stands for the amount of profit per two bushels.
This response receives 1 point for part A because correct design plan is provided. The support given in part A provides no indication of how the percentages were determined or to what they specifically pertain. In part B, the student provides an incorrect design plan and irrelevant support in context of what the item requires.

Score Point 1, Student Response 2
Part A. The best design plan I think would be the best for Mr. Fontenot is to plant 420 acres of soybean type A and then 105 acres of soybean type D. The reason i chose those two types of beans is while you may get less bushels per acre in type A the cost to harvest it is a considerably less amount than the other three. Type D on the other hand you can get 60 bushles per acre at a little higher harvest price than the others. **Part B** My system of 80 percent type A and 20 percent type B wasn't the worst choice about Mr. Fontenot's soybean harvest. It wold have ben better if it was 80 percent type D and 20 percent type C.
In part A, the student provides an incorrect plan and insufficient justification. The support provided indicates that the student only used values shown in the table and did not apply any further analysis. In part B, the student provides a correct design plan but offers no support for this conclusion. This response receives 1 point for the correct design plan in part B.

Score Point 1 for Minimal Understanding

A score of 1 for minimal understanding is given when a student has failed to receive points for any portion of the constructed-response item according to the rubric. Once a response receives 0 points according to the rubric, it is examined by the scorer to determine if the student has demonstrated minimal understanding of the standard being assessed. If the scorer determines that the student has demonstrated minimal understanding of the standard, then a response that received 0 points can earn a score of 1. Of the 750 student responses scored, none of the responses receiving 0 points demonstrated minimal understanding of the standard. In student response 1 for score point 0 on page 40 of this document, analysis is provided as to how that response could have received a score of 1 for minimal understanding if certain aspects of the response were slightly different.

Score Point 0

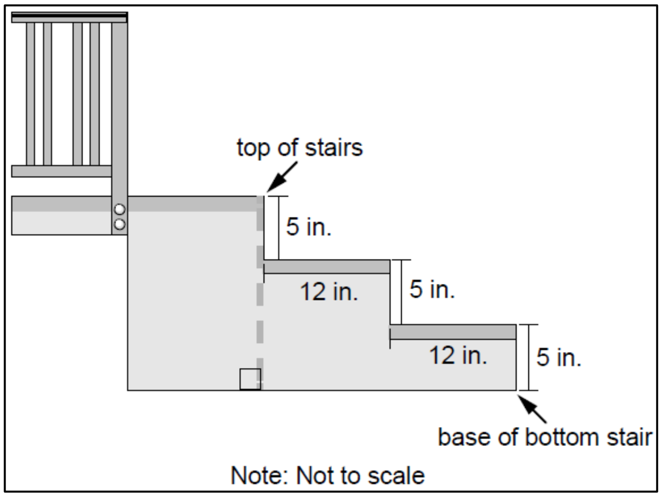
The following samples show the work of two students who each earned a score of 0. A score of 0 is received when a student response is incorrect, irrelevant, too brief to evaluate, or blank.

Score Point 0, Student Response 1
Part A I would plant Soybean type D and C. It cost only $3.40 to harvest one bush of type D and only $3.26 to harvest type C. I would plant mostly Saybean D becasue this will give you the max amount of profit after harvest and cost are deducted. i would Not plant a lot of type A because it cost $3.88 to harvest one bush. **Part B** Yes.If you planted 80% type D you would make a profit of about .84 on each bush. you would also make a profit of about .72 on type C bush. I was right by saying not to plant type B because you would only make a profit of about 0.27 on each bush harvested.
This response receives a score of 0. In part A, the student lists the two correct soybean types to plant but does not specifically state how much of each to plant, in fact, incorrectly stating that “mostly” soybean D should be planted. The support provided in part A is insufficient because it does not include cost per bushel for type B or comparative language such as highest and second highest. Had the student indicated that more of C should be planted than D, a score of 1 for minimal understanding may have been earned, showing not only that cost per bushel needs to be calculated to make a determination **but** also showing an understanding that a lower cost per bushel would result in lower harvesting costs. In part B, the student provides an incomplete design plan by not specifically stating which type should be planted on the other 20%. The student subtracted the cost per bushel from the profit per bushel which is irrelevant support for the design plan.

Score Point 0, Student Response 2
Part A "The best design plan for Mr. Fontenots 525 acre land is: Soybean A-300 acres Soybean B-150 acres Soybean C-75 ares 525-300=225 225-150=75 Soybean A is the most efficiant because Mr. Fontenot would be growing more crop for the lowest price, as for the other three soybeans, he would be overpaying per acre. **Part B** A-With 300 acres planted Mr. Fontenot would get 13,500. B-With 150 acres planted Mr. Fontenot would get 582 C-With 75 acres planted Mr. Fontenot would get 297 (other option besides A) C-With 300 accres planted Mr.Fontenot would get 1,188. Which is still less than plant A. B- with 300 acres planted Mr. Fontenot would get 1,164 None of the second choice options are as quiet efficiant as chosing A for more acres "
This response receives 0 points. In both parts, the student seems to have misunderstood the goals of the design plan by providing plans that involve planting 3 types of soybeans. The student does not provide relevant support for both parts.

GM: G-SRT.C.8 Building a Wheelchair Ramp
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Calculator Allowed

Use the diagram to answer the question.



Leah needs to add a wheelchair ramp over her stairs. The ramp will start at the top of the stairs. Each stair makes a right angle with each riser.

Part A

The ramp must have a maximum slope of $\frac{1}{12}$. To the nearest hundredth of a foot, what is the shortest length of ramp that Leah can build and not exceed the maximum slope? Show your work or explain your answer.

Part B

Leah decides to build a ramp that starts at the top of the stairs and ends 18 feet from the base of the bottom stair. To the nearest hundredth of a foot, what is the length of the ramp?

Part C

To the nearest tenth of a degree, what is the measure of the angle created by the ground and the ramp that Leah builds in part B?

This item requires students to reason about how slope, angle measures, trigonometric ratios, and the Pythagorean Theorem relate mathematically. Students must determine a horizontal measure for a leg of a right triangle that results in a hypotenuse slope of no more than $\frac{1}{12}$. Students then use the Pythagorean Theorem to find the measure of the hypotenuse in two different right triangles. Finally, students must use a trigonometric ratio to determine the measure of an angle in a right triangle. This item appears in the Online Tools Training (OTT).

Mathematical Practice(s)	
MP 1	Students must make sense of given information to determine how to solve each part of the problem. The problem calls on multiple banks of information and requires perseverance to solve each part.
MP 2	Students must decontextualize given measurements to manipulate the numbers using equations. Then, they must contextualize to determine the reasonableness of their answer within the given context.
MP 3	Students must construct an argument to justify their reasoning behind how they calculated the minimum length of ramp that will satisfy the given conditions in part A.
MP 4	Students use the Pythagorean Theorem to model the relationship between the legs and hypotenuse of the right triangle used to calculate the length of the ramp. Students use trigonometric ratios to model the relationship between side lengths and angle measures of the right triangle used to calculate the measure of the angle created by the ground and the ramp.

Scoring Information

This section includes information used to score this constructed-response item: an exemplary response, an explanation of how points are assigned, and a scoring rubric. Appropriate scoring parameters for all EOC constructed-response items are determined by Rangefinding Committees comprised of teachers and curriculum experts from across the state of Louisiana.

Scoring Rubric	
4	The student earns 4 points.
3	The student earns 3 points.
2	The student earns 2 points.
1	The student earns 1 point OR demonstrates minimal understanding of the standard being measured.
0	The student's response is incorrect, irrelevant to the skill or concept being measured, or blank.

Sample Answer

Part A.
15.05 feet
First find the length of the base of triangle (x)
1.25 ft/x = 1/12
1.25*12 = 1*x
X = 15
Then find hypotenuse/ramp (c)
1.25^2 + 15^2 = c^2
226.5625 = c^2 c =* 15.05

Part B
20.04 feet

Part C
3.6 degrees

Points Assigned

Part A
2 points:

- 1 point for correct length
- 1 point for complete and accurate work or explanation

Part B
1 point for correct length

Part C
1 point for correct degree measure

Note: The student may receive credit for part C for correctly finding the angle measure based on an incorrect value from part B.

Sample Student Responses[4]

Score Point 4

The following authentic student responses show the work of two students who each earned a score of 4. A score of 4 is received when a student completes all required components of the task and communicates his or her ideas effectively. The response should demonstrate in-depth understanding of the content objectives, and all required components of the task should be complete.

Score Point 4, Student Response 1
Part A 15.05 To find the shortest length of the ramp, I first set the stairs up as a right trianlge. The height of the triangle(base to top of stairs) was 15 inches so I used the maximum slope to set it up a proportion to find the length of the bottom of the triangle. I found the length would be 180 inches. I then used the pythagorean theorem to find the hypotenuse(length of the ramp) of the triangle. This answer was about 180.6324 inches. However, the length of the ramp length needed to be in feet so I converted it from inches to feet and got the shortest ramp possible would be 15.05 feet. **Part B** 20.04 **Part C** 3.6 degrees
This student response is correct and well-reasoned. The student calculates the correct length and provides a correct and complete explanation of his or her work for part A. The student also provides the correct length of the ramp in part B and the correct angle measure in part C.

Score Point 4, Student Response 2
Part A 15.05 feet 1/12 = 1.25/15 (1.25 x 1.25) + (15 x 15) = 226.56 sqrt(226.56 = 15.052 feet **Part B** 20.04 feet **Part C** 3.6
This response receives full credit. The student calculates the correct length and provides a correct and complete explanation of his or her work for part A. The student correctly calculates the length of the ramp in part B and the correct angle measure in part C.

[4] All student responses are authentic student work and not edited in any way, so responses may include typographical errors such as misspelled words or missing spaces.

Score Point 3

The following authentic student responses show the work of two students who each earned a score of 3. A score of 3 is received when a student completes 3 of the 4 components correctly. There may be simple errors in calculations or some confusion with communicating his or her ideas effectively.

Score Point 3, Student Response 1
Part A 15.05 feet The shortest length of ramp that Leah can build and not exceed the maximum slope is 15.05 feet. In order to get the length of the ramp, I first used the slope (1/12) as a ratio to find the length of the stairs. By multiplying the total heights of the risers, 15 inches, by 12, I found that the total length of the stairs would be 180 inches. By using the Pythagorean Theorum to find the length of the hypotenuse, I found that 15^2 + 180^2 =* 180.62^2. I then divided the hypotenuse by 12 to get the length of the ramp in feet. 180.62 / 12 = 15.05 feet. **Part B** 19.04 feet **Part C** 3.8 degrees
This student calculates the correct length and provides a correct and complete explanation of his or her work for part A. The student does not provide the correct length of the ramp in part B; however, the student's response in part C is correct based on this incorrect value in part B.

Score Point 3, Student Response 2
Part A 12.04 feet I added the rise of each step to find the length of the short leg. For each inch of the short leg I need 12 inches on the long leg. Then I used the pythagorean theorum to find the hypotenuse to find the length of the ramp. Last, I converted it to feet. **Part B** 20.04 feet **Part C** 3.6
In this response, the student provides an incorrect length of the ramp in part A. However, the student received partial credit of 1 point for providing a correct and complete explanation of how to determine the length of the ramp. The student correctly calculates the length of the ramp in part B and the correct angle measure in part C.

Score Point 2

The following authentic student responses show the work of two students who each earned a score of 2. A score of 2 is received when a student completes 2 out of 4 components correctly. There may be simple errors in calculations, one or two missing responses, or unclear or incorrect communications of his or her ideas.

Score Point 2, Student Response 1
Part A 15.05 I used the pythagorem thereom to get find that , with a slope of 1/12, each in down would take 12.04 inches. With this I could find the length of the ramp by multiplying it by the height,15 inches. Which came out to be 180.6 inches. Next, I needed to convert the answer to feet, which gave me 15.05 feet. **Part B** 18.04 **Part C** 6.90%
This student provides the correct length of the ramp in part A and correctly reasons that each vertical rise of 1 inch will result in a hypotenuse length of 12.04 inches and multiplies that length by the height. The student does not provide the correct length of the ramp in part B or the correct angle measure in part C.

Score Point 2, Student Response 2
Part A 15.05 feet I used a ratio and Pythagorean Theorem. Then I found how many feet. **Part B** 20.04 feet **Part C** 30 degrees
The student provides the correct length for the ramp; however, the explanation is incomplete. There is not enough detail to receive full credit for their reasoning. The student provides the correct length of the ramp in part B, but the angle measure provided in part C is incorrect.

Score Point 1

The following authentic student responses show the work of two students who each earned a score of 1 for their responses. A score of 1 is received when a student completes 1 out of 4 components correctly.

Score Point 1, Student Response 1
Part A 180.62 First I started out with what I know is going to be the height of the triangle we are going to make with the ramp and the floor. Since slope = rise/run I then figured out that the base of our tiangle would have to be 180in for the ratio to match 1/12. After that I simply used the pathagoream therom to figue out the hypotenuse or in this case, our ramp. **Part B** 216.52 **Part C** 4
This student provides the correct length of the ramp in part A, but expresses the measurement in the wrong unit. The item specifically asks for the length of the ramp in feet; however, the reasoning provided is correct and complete, so the student earned 1 point. The student does not provide the correct length of the ramp in part B or the correct angle measure in part C.

Score Point 1, Student Response 2
Part A 15 feet To the top of the stairs is 15 inches. So every inch you go up you have to go over one foot. Which would make the ramp 15 feet long. **Part B** 20 feet **Part C** 3.6 degrees
The student provides an incorrect ramp length. While the student correctly explains how to determine the measure of the long leg of the right triangle, the response is incomplete because the student does not provide a method for calculating the length of the hypotenuse. The student does not provide the correct length of the ramp in part B. However, the student's response in part C is correct based on this incorrect value in part B.

Score Point 1 for Minimal Understanding

Below is the work of students who earned a score of 1 for minimal understanding. Once a response receives 0 points according to the rubric, it is examined by the scorer to determine if the student has demonstrated minimal understanding of the standard being assessed. If the scorer determines that the student has demonstrated minimal understanding of the standard, then a response that received 0 points can earn a score of 1.

Score Point 1 for minimal understanding, Student Response 1
Part A 180.62 Because the ramp cannot exeed a slope of (1/12), the angle of the ramp cannot exeed 4.76 degrees. Using trigonometry, you can find the length of the ramp. **Part B** 44.6 **Part C** 19.65
This student provides the correct length of the ramp in part A in inches, but does not provide a unit of measure. The explanation is also incomplete as it does not provide a method for computing the hypotenuse of the right triangle. The student does not provide the correct ramp length in part B or the correct angle measure in part C. This response does not earn any points; however, one point is awarded for minimal understanding of the Pythagorean Theorem for providing the correct length of the ramp in part A using the wrong measurement unit.

Score Point 1 for minimal understanding, Student Response 2
Part A 39 in. A squared plus B squared equals C squared. 5 is A. 12 is B. 5 squared equals 25. 12 squared equals 144. 25 plus 144 equals 169. The square root of 169 is 13. There are three 5 inch drops. Multiply 13 in by 3 and that will equal 39 in. **Part B** 240.46 in. **Part C** 3.5
This student does not provide the correct ramp length in part A. The explanation does not take into account the slope of the ramp. While the Pythagorean Theorem is used correctly, it is applied using the wrong numbers. The student provides the correct ramp length in part B, but uses incorrect units of measure. The student provides an angle measure in part C that is a result of truncating the correct response instead of rounding to the nearest tenth. This response does not earn any points; however, one point is awarded for minimal understanding of using trigonometric ratios to solve problems for part C.

Score Point 0

The following samples show the work of two students who each earned a score of 0. A score of 0 is received when a student response is incorrect, irrelevant, too brief to evaluate, or blank.

Score Point 0, Student Response 1
Part A 12/13 12^2 + 5^2= 169 sqrt(169)= 13 rise/run 24/26 = 12/13 **Part B** 51 feet **Part C** 90 degrees
This student does not provide the correct length of the ramp in part A. The reasoning provided uses the Pythagorean Theorem, but it is applied using the wrong numbers. There is no mention of the slope of the ramp. The student does not provide the correct ramp length in part B or the correct angle measure in part C.

Score Point 0, Student Response 2
Part A 0.0ft If you divide 12 into 1 it comes out to be 0.083. round that to the nearest hundredth of a foot it comes out to be 0.1ft., anything less then 0.ft is 0.0ft. **Part B** 1.5ft **Part C** 2ft
This student does not provide the correct length of the ramp in part A. The reasoning provides no useful information for calculating the length of the ramp. The student does not provide the correct ramp length in part B. The answer provided for the missing angle in part C is incorrect in value and is expressed in the wrong unit of measure.

GM: G-MG.A.2 Determining Population Density

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Calculator Allowed

Use the table to answer the questions.

State Information

State	Population	Area (in square miles)
Louisiana	4,601,893	43,204
Mississippi	2,984,926	46,923
Rhode Island	1,050,292	1,034
Wyoming	576,412	97,093

Population density for a state is represented by the number of people per square mile.

Part A

Which of the states listed in the table has the greatest population density and what is that density? Round your answer to the nearest and make sure to include correct units.

Part B

A certain state has a population density that is higher than Louisiana. Describe the different ways the population and land area of this state could be the same or different from Louisiana.

This item requires students to calculate and compare population densities. Students are to observe patterns apparent in the work involved to complete part A as a basis for reasoning in part B.

Mathematical Practice(s)

MP 1	Students must examine and make sense of all of the given information in the problem and develop a solution pathway in order to provide the requested information.
MP 2	Students must use the given information and symbolic representations to contextualize their results and provide meaning for the quantities.
MP 6	Students must carryout the steps to solving with precision in order to determine the correct solutions. Any calculations shown must be free of mathematical errors.
MP 7	Students must compare the structure of differing divisors and dividends to their corresponding quotients to provide a basis for generalization about population density.

Scoring Information

This section includes information used to score this constructed-response item: an exemplary response, an explanation of how points are assigned, and a scoring rubric. Appropriate scoring parameters for all EOC constructed-response items are determined by Rangefinding Committees comprised of teachers and curriculum experts from across the state of Louisiana.

Scoring Rubric	
4	The student earns 4 points.
3	The student earns 3 points.
2	The student earns 2 points.
1	The student earns 1 point OR demonstrates minimal understanding of the standard being measured.
0	The student's response is incorrect, irrelevant to the skill or concept being measured, or blank.

Sample Answer

Part A

Rhode Island

1015.8 people per square mile

Part B

The state could have the same population as Louisiana, but less area. Or, the state could have a greater population, but the same area as Louisiana.

Points Assigned

Part A

2 points:

- 1 point for Rhode Island
- 1 point for the correct population density of Rhode Island with correct units

Part B

2 points:

- 2 points for 2 of the 3 possibilities: same area/higher population; less area/same population; less area/greater population

OR

1 point:

- 1 point for 1 of the 3 possibilities: same area/higher population; less area/same population; less area/greater population

There are no sample student responses for this constructed-response item.